Sixth Grade Mathematics

In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

(1) Students will use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students will connect their understanding of multiplication and division with ratios and rates. Thus students will expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students will solve a wide variety of problems involving ratios and rates.

(2) Students will use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students will use these operations to solve problems. Students will extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They will reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students will understand the use of variables in mathematical expressions. They will write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students will understand that expressions in different forms can be equivalent, and they will use the properties of operations to rewrite expressions in equivalent forms. Students will know that the solutions of an equation are the values of the variables that make the equation true. Students will use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students will construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they will use equations (such as $3x = y$) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students will begin to develop their ability to think statistically. Students will recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students will learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

(5) Students in Grade 6 will also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They will find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students will discuss, develop, and justify formulas for areas of triangles and parallelograms. Students will find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They will reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They will prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

Strand: THE NUMBER SYSTEM

Apply and extend previous understandings of multiplication and division of whole numbers to divide fractions by fractions.

1. Interpret and compute quotients of fractions.

   a. Compute quotients of fractions by fractions, *for example, by applying strategies such as visual fraction models, equations, and the relationship between multiplication and division, to represent problems.*

   b. Solve real-world problems involving division of fractions by fractions. *For example, how much chocolate will each person get if three people share 1/2 pound of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mile and area 1/2 square mile?*

   c. Explain the meaning of quotients in fraction division problems. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general (a/b) ÷ (c/d) = ad/bc.)*

Compute (add, subtract, multiply and divide) accurately, efficiently, and flexibly with multi-digit numbers and decimals and find common factors and multiples.

2. Accurately, efficiently, and flexibly divide multi-digit numbers using the standard algorithm.

3. Accurately, efficiently, and flexibly add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

   a. Accurately, efficiently, and flexibly divide multi-digit decimals using the standard algorithm, limited to a whole number dividend with a decimal divisor or a decimal dividend with a whole number divisor.

   b. Solve division problems in which both the dividend and the divisor are multi-digit decimals; develop the standard algorithm by using models, the meaning of division, and place value understanding.

4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two while numbers with no common factor. *For example, express 36 + 8 as 4(9 + 2)*.

Apply and extend previous understandings of numbers to the system of rational numbers.

5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (*for example, temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge*); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of zero in each situation.

6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

   a. Recognize opposite signs of numbers as indicating locations on opposite sides of zero on the number line; recognize that the opposite of the opposite of a number is the number itself. *For example, -(-3) = 3, and zero is its own opposite.*

   b. Understand that the signs of numbers in ordered pairs indicate their location in quadrants of the coordinate plane; recognize that when two ordered pairs different only by signs, the locations of the points are related by reflections across one or both axes.

   c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

7. Understand ordering and absolute value of rational numbers.

   d. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*

   e. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 °C > –7 °C to express the fact that –3 °C is warmer than –7 °C.*

   f. Understand the absolute value of a rational number as its distance from zero on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world context. *For example, for an account balance of –30 dollars, write* $|-30|$ *= 30 to describe the size of the debt in dollars*.

   g. Distinguish comparisons of absolute value from statements about order. *For example: Recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars..*

8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same *x*-coordinate or the same *y*—coordinate.

Strand: EXPRESSIONS AND EQUATIONS

Apply and extend previous understandings of arithmetic to algebraic expressions involving exponents and variables.

1. Write and evaluate numerical expression involving whole-number exponents.

2. Write, read, and evaluate expressions in which letters represent numbers.

a. Write expressions that record operations with numbers and with letters representing numbers. *For example, express the calculation "subtract y from 5" as 5 – y and express "Jane had $105.00 in her bank account. One year later, she had x dollars more. Write an expression that shows her new balance" as $105.00 + x.*

b. Identify parts of an expression using mathematical terms (*for example, sum, term, product, factor, quotient, coefficient*); view one or more parts of an expression as a single entity and a sum of two terms. *For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, applying the Order of Operations when there are no parentheses to specify a particular order. *For example, use the formulas* $V = s^3$ *and* $A = 6s^2$ *to find the volume and surface area of a cube with sides of length* $s = 1/2$.

3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y*.

4. Identify when two expressions are equivalent. *For example, the expressions y + y + y and 3y are equivalent because they name the same number, regardless of which number y represents*.

Reason about and solve one-variable equations and inequalities.

5. Understand solving an equation or inequality as a process of answering the question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or depending on the purpose at hand, any number in a specified set.

7. Solve real-world and mathematical problems by writing and solving equations of the form $x + a = b$ and $ax = b$ for cases in which a, b and x are all non-negative rational numbers.

8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables in a real-world context.

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Strand: RATIOS AND PROPORTIONAL RELATIONSHIPS

Understand ratio concepts and use ratio reasoning to solve problems.

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. The following are examples of ratio language: "*The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every two wings there was one beak.*" "*For every vote candidate A received, candidate C received nearly three votes*."

2. Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with *b* ≠ 0, and use rate language in the context of a ratio relationship. The following are examples of rate language: "*This recipe has a ratio of four cups of flour to two cups of sugar, so the rate is two cups of flour for each cup of sugar*." "*We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger*." (In sixth grade, unit rates are limited to non-complex fractions.)

3. Use ratio and rate reasoning to solve real-world (with a context) and mathematical (void of context) problems, using strategies such as reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations involving unit rate problems.

   a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

   b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took four hours to mow eight lawns, how many lawns could be mowed in 32 hours? What is the hourly rate at which lawns were being mowed?*

   c. Find a percent of a quantity as a rate per 100. Solve problems involving finding the whole, given a part and the percent. (*For example, 30% of a quantity means 30/100 times the quantity.)*

   d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Strand: GEOMETRY

Solve real-world and mathematical problems involving area, surface area, and volume.

1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing and decomposing into rectangles, triangles and/or other shapes; apply these techniques in the context of solving real-world and mathematical problems.
2. Find the volume of a right rectangular prism with appropriate unit fraction edge lengths by packing it with cubes of the appropriate unit fraction edge lengths *(for example, 3 1/2 × 2 × 6)*, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. (Note: Model the packing using drawings and diagrams.)
3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same x coordinate or the same y coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Strand: STATISTICS AND PROBABILITY

Develop understanding of statistical variability of data.

1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in the students' ages*.
2. Understand that a set of data collected to answer a statistical question has a distribution that can be described by its center, spread/range and overall shape.
3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

4. Display numerical data in plots on a number line, including dot plots, histograms and box plots. Choose the most appropriate graph/plot for the data collected.
5. Summarize numerical data sets in relation to their context, such as by:
   a. Reporting the number of observations.
   b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations (*for example, outliers*) from the overall pattern with reference to the context in which the data were gathered.

d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.